Edge Current of FQHE and Aharanov-Bhom Type Phase

Dipti Banerjee
Department of Physics
Rishi Bankim Chandra College
Naihati,24-Parganas(N)
Pin-743 165, West Bengal
INDIA

Abstract

When two non-identical quasi-particles in the Hall fluid encircle each other, relative AB type phase developes.As the quasi-particles advance towards the edge in a similar circular way, the developed current should have connection with this AB type phase through the *Shift* quantum number or Berry's topological phase. We have pointed out the role of relative AB type statistical phase in the development of edge current.In fact,the physics of the current flow in FQHE is sketched here from the topological point of phase transformation.

email:deepbancu@hotmail.com

1 Introduction

A fundamental feature of the microscopic theory is the existence of a one-to-one correspondence between the quasi particle states of the bulk and the primary fields that build the spectrum of the edge states.Microscopically the bulk states can be constructed by implementing the idea of flux-attachment by coupling suitable set of Chern-Simon gauge fields[1]. The effective action in the field theory is a Chern-Simon gauge theory that fits the K-matrix classification [2] reflecting the underlying hierachical construction of FQH states. The composite picture of the edge states have number of branches that is encoded in the rank of the K-matrix. Wen has shown that edge excitations in FQH states provide an important probe to detect the topological orders in the bulk FQH states.

At the edge, the electrostatic potential varies very slowly and adiabatic conductance takes place between series of alternating compressible and incompressible strips forming channels at the edge. The QH liquid for $\nu = 1/m$ contains only one component of incompressible state that lead to one branch. A generic QH state with $\nu \neq 1/m$ contains many branches of edge excitations. The concept of edge channels is extended from the integer to the fractional quantum Hall effect, and the contribution of an adiabatically transmitted edge channel to the conductance is calculated from the point of view of interacting-electron picture [3]. For a sufficiently small $\Delta\mu$ (chemical potential) the current carried by quasi-particles in a compressible region at the edge depends on the on the difference of the electron filling factors in the two adjacent incompressible regions[4].

It is well known that these quasi-particles in FQHE are not fundamental particles and obey fractional statistics in two dimensions. Any fractional statistics objects are collective particles of a nontrivial condensed matter state. The fractional statistics as pointed out by Leinaas and Myrheim [5] relies on the property that when particles with infinitely strong short range repulsion are confined in two dimensions, paths with different winding numbers are topologically distinct and cannot be deformed into one another. The particles [6] are said to have statistics θ producing a phase factor $e^{i\pi\theta} = (-1)^{\theta}$ when exchange of particles takes place over a half loop. Non integral values of θ imply fractional statistics.

It is believed [7] that fractional statistics are the consequences of incompressibility at a fractional filling and may possible be observable in an experiment specially designed for this purpose. The fractional statistics can be derived heuristically in the Composite Fermion theory [8] when one CF goes around another encircling an area A, the total phase associated with this path is given by

$$\Phi^* = -2\pi(BA/\phi_0 - 2pN_{enc})$$

where N_{enc} is the number of composite fermions inside the loop. The first term on the right-hand side is the usual AB phase and the second term is the contribution from the vortices bound to composite fermions indicating that each enclosed CF effectively reduces the flux by $2p$ flux quanta. Wilczek shed considerable light on fractional statistics of quasi particles [9].In two dimensions these quasi-particles are

similar as vortex attached at the point particle. If a vortex is dragged adiabatically around a closed loop,the system will acquire an extra non dynamical phase which can be gauged away resulting continuously from one incompressible liquid state to another at different filling fraction. The concept of fractional statistics has been reformulated by Haldane [10] as a generalization of Pauli exclusion principle and a definition independent of the dimension of space. In the FQHE, the Pauli like definition of statistics can be introduced in the quasi particles which are flux carrying charged bosons in the lowest Landau level.If an object carrying flux ϕ_α and charge q_α orbits around another object carrying flux ϕ_β and charge q_β the relative statistical phase $\theta_{\alpha\beta}$ becomes

$$exp(i\theta_{\alpha\beta}) = exp \pm i\pi(g_{\alpha\beta} + g_{\beta\alpha}) \tag{1}$$

where

$$g_{\alpha\beta} = -q_\alpha \phi_\beta$$

With these view we have recently shown [11] that due to interchange of two non-identical composite fermions residing in two consecutive Landau levels,the relative AB type phase developed and *shift* quantum number can be visualized through it. In another issue [12] we have showed that this *shift* vector have connection with the change of edge current through the difference of filling factor $\nu_n - \nu_{n-1}$ between two consecutive incompressible states of the edge. In fact a more meaningful idea of the *shift* [13] has been given through deviations of Berry's topological phase of composite particles in hierarchies.

Motivated by the recent two works, one which highlights the importance of non equilibrium noise measurement through statistical phase [14] and other is FQHE qubits in connection with topological quantum computation [15], we realize that in the transport process of edge current, the relative AB type phase should play a major role. Hence we here want to find the origin of the edge current developed through the statistical interaction of composite particles in the AB type topological phase.

2 Topological Aspect of Hall fluid, Berry Phase and Shift quantum number

In the Hall fluid the statistical interaction takes the most significant role. Being long ranged it is treated non-perturbatively. A non-dynamical gauge field A_μ is associated with the flux which in $2+1$ dimension is the very cause of the appearance of Chern-Simon term in the Lagrangian.

$$L_{CS} = \frac{\mu}{2}\epsilon^{\mu\nu\lambda} A_\mu \partial_\nu A_\lambda \tag{2}$$

This licenses a conservation of topological current J_μ which include a topological invariant term in the $(2+1)$ dimension [9]

$$H = \frac{\theta}{2\pi}\int d^3x A_\mu J^\mu \tag{3}$$

in the action. In fact it is the Hopf invariant describing basic maps of S^3 to S^2. If ρ denotes a four dimensional index then we find

$$\partial_\rho \epsilon^{\rho\mu\nu\lambda} A_\mu F_{\mu\nu} = \frac{1}{2} \epsilon^{\rho\mu\nu\lambda} F_{\rho\mu} F_{\nu\lambda} \tag{4}$$

which connects the Hopf invariant with chiral anomaly. This Hopf term plays a role somewhat similar to the role played by the Wess-Zumino interaction in connection with $3+1$ dim. Skyrmion term.

There is an analogous statistical interaction in (3+1) dimensions given by Haldane [16] considering the 2D Hall surface as a boundary surface of a 3D sphere, having radius R in a radial (monopole) magnetic field $B = \hbar S/eR^2 (> 0)$. This $2S = N_\phi$ is an integer which defines the total number of magnetic flux through the surface. For the parent state $\nu = 1/m$ the total flux is $S = \frac{1}{2} m(N-1)$. The field strength S in the first level hierarchy is

$$S(N, m \pm p) = \frac{1}{2} m(N-1) \pm \frac{1}{2} (\frac{N}{p} + 1)$$

which is formed when p (p= even integer) excitations are added in the parent state $\nu = \frac{1}{m}$. These show that the filling factors for hierarchical state satisfy a slight complicated relation.

$$2S = N\phi = \nu^{-1} N - \mathcal{S}$$

In the language of Wen and Zee [1,2] this $\mathcal{S}$ is the *shift*, a topological quantum number which is developed due to the coupling between the orbital spin and the curvature of the space having spin $s = \frac{1}{2} K_{II}$. On a sphere,the *shift* for a hierarchical state is given by

$$\mathcal{S} = \frac{1}{\nu} \sum_{IJ} (K^{-1})_{IJ} K_{JJ} \tag{5}$$

For a $\nu = \frac{1}{m}$ parent state this *shift* is simply $\mathcal{S} = 2(n-1) + m$ having orbital spin $s = n - 1 + \frac{m}{2}$ that is associated with the orbital angular momentum in cyclotron motion. In the effective theory, this introduction of *shift* leads to a modification of the Lagrangian in equation (2) as follows

$$\mathcal{L} = 1/4\pi (KB\epsilon\partial B + 2Ae\epsilon\partial B + 2Cs\epsilon\partial B) \tag{6}$$

where the second term is the electromagnetic coupling and the third one is the coupling to the curvature of space.

The appearance of *shift* in the hierarchies of FQHE is nontrivial [13]. The quasiparticles in these levels are formed when additional fluxes are attached with quantized particles. In fact the quantization of Hall particles is the indication of Quantum Hall effect involving gauge theoretic extension of coordinate by $C_\mu \in SL(2C)$ which is visualized through the field strength $\tilde{F_{\mu\nu}}$. Apart from the internal extension, the external strong magnetic field induces gauge extensions $B_\mu \in SL(2C)$ through the gauge field $F_{\mu\nu}$.In the language of differential geometry these two gauges

act as two fibres at each particle points of the base space S^2. The effective theory of the Hall fluid (Abelian) can be accurately presented if not only the two vortices but also their interactions are taken into account. In the light of Haldane[16], we consider the Hall surface on 3D sphere and in the presence of strong external magnetic field, the chiral symmetry breaking of composite fermion associated with internal and external gauge fields $F_{\mu\nu}$ and $\tilde{F_{\mu\nu}}$ are represented by

$$\begin{aligned} F_{\mu\nu} &= \partial_\mu B_\mu - \partial_\nu B_\nu + [B_\mu, B_\nu] \\ \tilde{F_{\mu\nu}} &= \partial_\mu C_\mu - \partial_\nu C_\nu + [C_\mu, C_\nu] \end{aligned} \tag{7}$$

In particular the θ term in the Lagrangian leads to vortex line and the corresponding gauge field acts like a magnetic field. The topological Lagrangian of Hall fluid can be described by the added Chern-Simon terms in the Lagrangian through the anomaly [13]

$$\mathcal{L} = -\frac{\theta}{16\pi^2} Tr^* F_{\mu\nu} F_{\mu\nu} - \frac{\theta'}{16\pi^2} Tr^* F_{\mu\nu} \tilde{F}_{\mu\nu} - \frac{\theta''}{16\pi^2} Tr^* \tilde{F}_{\mu\nu} \tilde{F}_{\mu\nu} \tag{8}$$

Here every term corresponds to a total divergence of a topological quantity, known as Chern-Simons secondary characteristics class defined by

$$\Omega^\mu{}_e = -\frac{1}{16\pi^2 \epsilon_{\mu\nu\alpha\beta}} Tr[B_\nu F_{\alpha\beta} - 2/3(B_\nu B_\alpha B_\beta)] \tag{9}$$

$$\tilde{\Omega}^\mu = -\frac{1}{16\pi^2 \epsilon_{\mu\nu\alpha\beta}} Tr[C_\nu F_{\alpha\beta} - 2/3(C_\nu B_\alpha B_\beta)] \tag{10}$$

$$\Omega^\mu{}_i = -\frac{1}{16\pi^2 \epsilon_{\mu\nu\alpha\beta}} Tr[C_\nu \tilde{F}_{\alpha\beta} - 2/3(C_\nu C_\alpha C_\beta)] \tag{11}$$

Assuming a particular choice of coupling $\theta = \theta' = \theta$" in the Lagrangian the topological part of the action in (3+1) dimension become

$$W_\theta = 2(\mu_e + \mu_i + \tilde{\mu})\theta \tag{12}$$

where μ_e, μ_i and $\tilde{\mu}$ are the corresponding magnetic charges which are connected with the respective charges through the Dirac quantization condition and Pontryagin density.

$$2\mu = q = \int \partial_\mu \Omega_\mu d^4x \tag{13}$$

It gives rise the topological phase of Berry on the parallel transport over a closed path of a Hall hierarchy state.

$$\phi_B = \pi W_\theta = 2\pi \tilde{\mu}_{eff} \theta = 2\pi(\mu_{eff} + \tilde{\mu})\theta = 2\pi(\mu_e + \mu_i + \tilde{\mu})\theta \tag{14}$$

Here the first term is associated with Berry phase factor of Hall particle due to external magnetic field μ_e. The second term gives rise to the inherent Berry phase factor μ_i associated with the chiral anomaly of a free electron (in absence of an

external magnetic field) and the third one effectively relates the coupling of the external field with the internal one which give rise the phase factor $\tilde{\mu}$. This μ_{eff} actually visualizes the filling factor through the relation $\nu = \frac{n}{2\mu_{eff}}$. where n denotes the nth Landau level. In fact this μ_{eff} satisfies the Dirac quantization condition

$$e'\mu_{eff} = \frac{n}{2} \tag{15}$$

showing that each quasi particle in the n^{th} Landau level having charge e' behaves as a composite fermion. It will behave as fermion in the ground state following the Dirac condition

$$\tilde{e\mu} = \pm 1/2 \tag{16}$$

which follows the equation

$$\tilde{e}(\mu_{eff} - \frac{n \pm 1}{2}) = \pm\frac{1}{2} \tag{17}$$

This implies that $(n \pm 1)/2$ is the magnetic strength μ of the added quanta whose removal makes the composite fermion in the higher Landau levels to behave fermion in the ground state. Here for $\mu = \pm 1/2, \pm 3/2, ...$ the quanta behave like fermion and $\mu = \pm 1, \pm 2, ...$ it shows bosonic behavior. We have found this change of magnetic charge as $\tilde{\mu}$ that can be visualized through *shift* $\mathcal{S}$ by the relation

$$2\tilde{\mu} = \mathcal{S} = 2\mu_{eff} - (n \pm 1) = \frac{n}{\nu} - (n \pm 1) \tag{18}$$

where $n = 1, 2, 3..$ denotes the hierarchy levels.

Our picture shows that a motion of composite particle in the Hall fluid moving in a circular orbit will be quantized through its acquirance of the Berry's topological phase.

$$\phi_B = \pi\theta(2\mu_{eff} + \mathcal{S}) \tag{19}$$

Conceptually the appearance of this *shift* quantum number $\mathcal{S}$, in the topological phase of quasi-particle is obvious, since the coupling between the two gauges(that act as fibres) with the curvature is prominent during parallel transport over a closed path.

With this view we have found [11]the role of *shift* in the relative AB type phase as the composite fermion and the additional quanta encircles each other for producing fermions in the lowest Landau level. In addition, we have shown elsewhere [12] in the context of edge current flowing through the compressible level that this *shift* can be related with the difference of Landau filling factor between two consecutive incompressible levels.

$$\frac{\tilde{e}}{2\mu_eff}\mathcal{S} = (\nu_n - \nu_{n-1}) \tag{20}$$

For a sufficiently small chemical potential $\Delta\mu$, the change of current in a compressible strip is,

$$\Delta I = \frac{e}{h}\Delta\mu\Delta\nu = \frac{e}{h}\Delta\mu(\nu_n - \nu_{n-1}) \tag{21}$$

that can be expressed in terms of *shift*

$$\Delta I = -\frac{e}{h}\Delta\mu\frac{\tilde{e}}{2\mu_e ff}\mathcal{S} \tag{22}$$

Now we will proceed to find the role of A-B type statistical phase in the development of edge current as the composite particles advance on the Hall surface.

3 The Edge current in A-B Type Phase

The concept of edge channels for IQHE and FQHE in combination with the adiabatic transport of quasi-particle is successful in explaining the anomalous dependence of Hall conductance. Edge channels are defined with the correspondence of bulk landau level.On approaching the boundary of the 2DEG a Landau level which in the bulk lies below the Fermi level rises in energy because of the presence of confining potential. The intersection between the nth Landau level and the Fermi level defines the location of the nth edge channel for filling factor in the nth hierarchy. In general the current injected into *pth* edge channel is [3]

$$I_p = \frac{e}{h}\Delta\mu(\nu_p - \nu_{p-1}) \tag{23}$$

where the current I_p in a compressible band is in between two incompressible bands of filling factors ν_p and ν_{p-1}.

The tunnelling current I through the wire is $I(V) \propto V^{\alpha}$, where the exponent α is determined by the scaling dimension of the tunnelling operator. Lopaz and Fradkin [17] pointed out recently that in case of tunnelling of electrons from Fermi liquid into a hierarchical FQH state, the tunnelling exponent is $\alpha = 1/\nu$. The physics behind the tunnelling in the edge has been focussed on the charge and neutral modes that are propagating with different velocities. This latter one has been identified as topological mode which is responsible for Fermi statistics.Representing the respective charge mode and topological mode by ϕ_c and ϕ_T, a general edge operator is

$$\psi(x) = expi(\alpha_c\phi_c + \alpha_T\phi_T + \Sigma\alpha_T\phi_T) \tag{24}$$

The authors have shown that charge Q depend only on α_c but the statistics θ of these excitations is connected with both α_c and α_T.

In a recent communication Zulicke and Mac Donald et.al.[18] addressed the chiral phase field $\phi_n(\theta)$ as a superposition of edge-density fluctuations.

$$\phi_n(\theta) = \frac{1}{\sqrt{\nu}}\phi^c(\theta) + \xi_n\phi^n(\theta) \tag{25}$$

where ϕ^c is the phase field of the charged edge-magneto plasmon mode which corresponds to fluctuations in the total edge-charge density and ϕ^n is its orthogonal complement known as neutral mode.The authors have expressed these two modes

in terms of the parent state ϕ_0 and daughter state ϕ_{2p+1} with the respective filling factors $\nu_0 = \frac{1}{2p+1}$ and $\nu_i = \frac{1}{(2p+1)(4p+1)}$, that comprise the $\nu = \frac{2}{4p+1}$ QH state. The addition of electrons to the edge with concomitant change of $2p+1-n$ flux quanta is viewed as adding the electron to the outer edge and transferring at the same location n fractionally charged quasiparticles from the outer QH droplet to the inner one.

We realize from the above works that both the charge and neutral modes are transferred from the inner edge to outer edge leading to flow of current and change of statistics. We are now interested whether this current and statistics are interrelated during the course of transfer towards the edge. Inspired by the works on topological transformation of current in the FQHE system in connection with Quantum Computation [15] through fractional statistics, we now proceed to evaluate the role of AB type Statistical phase in edge current flow two ways.

1. In a particular edge the composite particles in the consecutive branch (Landau level) encircles during transformation.
2. From inner edge the composite particles transform to the outer edge picking up integral multiple of flux from the bulk.

It is now known that composite particles in FQHE are the composite of fluxes attached with charged particle.When an electron is attached with a magnetic flux, its statistics changes and it is transformed into a boson.These bosons condense to form cluster which is coupled with the residual fermion or boson (composed of two fermions). Indeed the residual boson or fermion will undergo a statistical interaction tied to a geometric Berry phase effect that winds the phase of the particles as it encircles the vortices. Also we observe that the attachment of vortices to electrons in a cluster will make the fluid an incompressible one. Indeed as two vortices cannot be brought very close to each other, there will be a hard core repulsion in the system which accounts for the incompressibility of the Quantum Hall fluid. In fact the Hall particles are quantized by acquiring Berry's topological phase as discussed in sec.-2. As the quasi-particles encircles another in their way of topological transport, the Aharanov-Bhom type statistical phase is developed.

At first, we concentrate on one edge of a QH system where we find current in the compressible band (eqn.-23) depends on the filling factors of the consecutive incompressible Landau levels n^{th} and $(n-1)^{th}$ respectively. During this movement of quasiparticles, the charge dressed with flux advance following circular path.As one encircles another relative AB type phase developed [10]

$$\phi_s = exp \pm \frac{i\pi}{2}(q_n\mu_{n-1} + q_{n-1}\mu_n) \tag{26}$$

where q_n, q_{n-1} are the respective charges and μ_n, μ_{n-1} are the corresponding magnetic strength of the flux attached.These composite particles follow Dirac Quantization condition $q_n\mu_n = n/2$ having respective filling factors $\nu_n = \frac{n}{2\mu_n}$ and $\nu_{n-1} = \frac{n-1}{2\mu_{n-1}}$. Now the intertwining of these Composite particles against each

other results

$$\phi_s = exp\pm\frac{i\pi}{2}(\nu_n\mu_{n-1} + \nu_{n-1}\mu_n) \tag{27}$$

$$\phi_s = exp\pm\frac{i\pi}{2}(\frac{{\nu_n}^2(n-1) + {\nu_{n-1}}^2 n}{2\nu_{n-1}\nu_n}) \tag{28}$$

After a few mathematical steps and using eqn.-23 we have

$$\phi_s = exp \pm \frac{i\pi}{2}(\frac{nK^2I^2}{2\nu_{n-1}\nu_n} + 2n - \frac{\nu_n}{2\nu_{n-1}}) \tag{29}$$

where $K = \frac{e}{h}\Delta\mu$.
This implies that edge current or its change can be realized through the acquirance of AB type statistical phase whenever two quasi-particles in the consecutive Landau level encircle each other.In other words the noise in the current flow is the very cause of these type of phase factor.

In the second case,we consider the edge tunnelling through the bulk of FQHE. We assume the transfer of the composite particle from the inner edge in the n^th Landau level having filling factor ν_n picking up even integral $(2m)$ of flux ν_1 through the bulk of QH system and forming a new composite particle in the $(n+1)^th$ Landau level of the outer edge. The filling factor of the effective particle becomes $\nu_{eff} = \frac{n+1}{\mu_{eff}}$.In the light of Haldane [16] and Jain [19], we consider that the monopole strength μ_{eff} of the state ${\Phi_1}^{2m}\Phi_n$ can be obtained by noting that the product of two monopole harmonics μ_1 and μ_n gives a monopole harmonic at $\mu_1 + \mu_n$ i,e monopole strength add as follows

$$\mu_{eff} = 2m(\frac{N-1}{2}) + \frac{N-n^2}{2n} \tag{30}$$

which can be considered as

$$\mu_{eff} = 2m\mu_1 + \mu_n. \tag{31}$$

Here statistical interaction takes place between the composite particle of the inner edge and outer edge which result current propagation. We further assume that path of the particles do not intersect each other. Encircling one type of fluxes around another in the consecutive Landau level relative AB type phase produces that is the very cause of edge current flow.

Following Haldane [10], we consider encircling of the composite particle in the inner edge having flux μ_n with charge q_n that is equivalent to the filling factor $\nu_n = \frac{n}{2\mu_n}$, around the composite particle in the outer edge having corresponding flux and charge respectively μ_{eff} and

$$q_{eff} = \frac{n+1}{2\mu_{eff}} = \frac{n+1}{2m\mu_1 + \mu_n}$$

. This generate the relative AB type phase developed by their fluxes and charges as

$$\phi_s = exp \pm \frac{i\pi}{2}(q_n\mu_{eff} + q_{eff}\mu_n) \tag{32}$$

Since the quasiparticles satisfy the Dirac quantization relation we can write the above equation as

$$\phi_s = exp \pm \frac{i\pi}{2}(\nu_n\mu_{eff} + \nu_{eff}\mu_n) \tag{33}$$

After a few mathematical steps we found similar equation (as eq.-29) of the relative AB type statistical phase

$$\phi_s = exp \pm \frac{i\pi}{2}(\frac{nK^2I^2}{2\nu_{eff}\nu_n} + 2n - \frac{\nu_n}{2\nu_{eff}}) \tag{34}$$

developed due to the transfer of a composite particle from inner edge to the outer edge through the bulk carrying the integral multiple of flux alongwith. We see that we get identical result in our two different approaches of edge current flow.

From another point of view using equation 31 we have

$$\phi_s = exp \pm \frac{i\pi}{2}(\frac{n}{\mu_n}(2m\mu_1 + \mu_n) + \frac{(n+1)\mu_n}{(2m\mu_1 + \mu_n)} \tag{35}$$

$$\phi_s \cong exp \pm \frac{i\pi}{2}[\frac{n}{2}(2m\frac{\mu_1}{\mu_n} + 1) + \frac{n+1}{2}(1 - 2m\frac{\mu_1}{\mu_n})] \tag{36}$$

$$\phi_s \cong exp \pm \frac{i\pi}{2}[(n + \frac{1}{2}) - m\frac{\mu_1}{\mu_n}] \tag{37}$$

With our previous knowledge we see that this phase factor has relationship with *shift* quantum number. Above all we can comment from the above equation that irrespective of μ_1 and μ_n being fermionic or bosonic flux, the phase factor depends upon the number of particles-N,the Landau level-n, and the odd integer-m that is the inverse of parent filling factor $\nu = 1/m$ by the following expression.

$$\phi_s \cong exp \pm \frac{i\pi}{2}[(n + \frac{1}{2}) - \frac{mn(N-1)}{(N-n^2)}] \tag{38}$$

Recently it has been found that fractional statistics play an important role in topological transformation in connection with Quantum computation [15]. Also Kane [14] showed that statistical phase by combining AB effect can be used in noise measurement. The result we find fully support these views. The current obtained from the contacts of the Hall edge considering the effect of bulk also can be visualized through AB type quantum phase.

Discussions

Quantization of Hall particles in the hierarchical states ensure the acquirance of Berry's topological phase that visualize the resultant chirality of the hierarchical state. Interchange of two identical quasi particles develop statistical phase. Whereas two dissimilar quasi particles on encircling each other produces relative AB type phase. In this paper, we shed light on the latter phase for its role in the edge current flow. At the edge, nonzero current appears in the hierarchies which originate

from the non-vanishing anomaly in terms of the deviation of the topological phase through the difference of filling factors. The quasi particles responsible for current flow, encircle one around other in course of their advancement towards edge of Hall surface. As a result relative AB type statistical phase evolved for the intertwining of fluxes around the charges of the quasi particles in the following two cases.
1. In a particular edge the quasi-particles in the consecutive branch (incompressible Landau level) encircles during transformation.
2. From inner edge the quasi-particles flow to the outer edge picking up integral multiple of flux from the bulk of the Hall system.
We find that in both the cases AB type statistical phase is directly connected with edge current. And the second case combine the physics of the edge and the bulk for the current flow. Hence classical current can be visualized through quantum phase. In future these findings will help us to work for *Quantum Computation with FQHE qubits*[13] and *Spin propagation in the Spintronics* devices [20].

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